Farsighted Free Trade Networks

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Abstract

The paper examines the strategic stability and the efficiency of free trade networks. We reconsider the endogenous tariff model introduced by Goyal and Joshi (2006). Different from their analysis with myopic countries, we adopt a new solution concept, *pairwise farsighted stable set (*Herings, Mauleon and Vannetelbosch, 2009, GEB), to examine the formation of free trade agreements (FTAs) as a network formation game among farsighted countries. Our principal finding shows that, the global free trade network is pairwise farsightedly stable, confirming the consistency between bilateralism and multilateralism. We also explore the efficiency and uniqueness of the complete global free trade network.

1 Introduction

Even though multilateral trade negotiations (described here simply as "multilateralism") are supported as the main mechanism for encouraging free trade, the surge of forming regional trade agreements (RTAs) has continued unabated since the early 1990s. As of Dec. 2008, 410 RTAs have been notified to the GATT/WTO, of which, free trade agreements (FTAs) and partial scope agreements account for over 90%, while customs unions account for less than 10%.[1] With the growing popularity of Free Trade Areas (FTAs), considerable attention has been focused on whether FTAs are conducive or detrimental to globalization. On the one hand, FTAs offer a quicker and surer way of getting to free trade whereas multilateralism is too slow and inefficient in getting there, but on the other hand, FTAs are in conflict with the Most Favored Nation (MFN) principal enunciated in GATT Article I. Realizing RTA is a double-edged sword, on the 6th, February 1996, the WTO created the Regional Trade Agreements Committee whose purpose is to examine regional groups and to assess whether they are consistent with WTO rules.

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[1] See www.wto.org. Free trade agreements (FTAs) and customs unions are two forms of RTAs. FTAs are more commonly occurred than customs unions.

The present paper examines the impacts of bilateral FTAs on trade liberalization. In particular, we study the formation of bilateral FTAs as a network formation game among farsighted countries. Two related issues are investigated in this paper. The first one is focused on which free trade network is socially efficient, while the second one is concerned with whether or not this efficient free trade network could be sustained when each forward-looking country establishes bilateral free trade agreements voluntarily. We utilize the endogenous tariff model developed by Goyal and Joshi (2006). There are n countries. In each country there is a single firm who can sell in both domestic and foreign markets. These countries play a three stage game. In the first stage, countries can sign bilateral FTAs, and a collection of FTA links defines a trading regime; In the second stage, all countries set optimal tariff rates on those countries who do not establish FTA links with them. At the same time, the countries who are involved in bilateral FTA links have free access to each other's markets; In the third stage, all firms compete as Cournot oligopolists in each country's market. Goyal and Joshi (2006) firstly introduced network formation game to the study of trading regime.[2] In their setting, they assume all countries are myopic. Then in the process of FTA network formation, all countries form and sever links based on the improvement of the resulting network offers them relative to the current network. This formation process may end at *pairwise stable* networks.[3] However, with myopic consideration, it is possible that a country suffers from deleting a link, but this deletion leads another country to add another link which in turn leaves the first country better off relative to the starting position. If a forward-looking country predicts this, it should choose to remove the link to initiate the process. Unfortunately, this farsighted behavior is not taken into account in the existing literature dealing with free trade network formation games. To capture the feature of stable networks among farsighted countries, we adopt a new solution concept, *pairwise farsightedly stable set,* proposed by Herings, Mauleon and Vannetelbosch (2009). In this dynamic process, all countries form and sever links based on the improvement that the end network offers relative to the current network. This dynamic process will end at networks in a pairwise farsightedly stable set.

The main results of this paper are as follows. In a setting with n symmetric countries, the global complete free trade network is the unique strongly efficient network (Proposition 3). This implies that the total welfare of all countries is maximized in the world trading system. Furthermore, a dynamic free trade network formation process is considered. With the assumption of forward looking countries, the complete global free trade network is able to be sustained as a stable outcome. Specifically, the complete global free trade network, as the only strongly efficient network, is pairwise farsightedly stable (Proposition 8). This proposition suggests that starting from any trading configuration, as long as countries are forward-looking, global free trade can be achieved through a series of farsighted decisions made by countries. However, the complete network is not the unique pairwise farsightedly stable one (Proposition 9). Some other inefficient networks are also sustained as stable outcomes. This multiplicity of stable outcomes shows the dynamic free trade network formation

[2] Independently, Furusawa and Konishi (2005, 2007) also use network formation models to study the international trading system.

[3] The notion of pairwise stability is introduced by Jackson and Wolinsky (1996). A network is pairwise stable if no player gains from severing one of their links and no any two players benefit from establishing a link between them.

process is "path dependent." Some paths lead the trading system to the global free trade while others might lead the process to other inefficient trading regimes.

Related literatures

Whether bilateral free trade agreements (FTAs) help or hinder the world trading system has been a long-standing question. In recenct years, there has been a large body of literature dealing with the compatiblity between bilateralism and multilateralism. We now relate our paper to the existing studies on this issue. According to Bhagwati (1993), regional trade agreements could be "stumbling blocs" hindering the process of global trade liberalization, or "building blocs" facilitating the achievement of worldwide free trade. Many trade economists (like Bhagwati 1993, Krugman 1993) have forcefully prompted that the regional agreements impede multilateral agreements. In this view, allowing for the freedom to pursue FTAs would affect the achievement of global free trade. Such preferential free trade agreements will generate welfare gains for member countries so that each of them reduce the incentive to seek further trade liberalization. For example, Levy (1997) considered a median-voter model in a monopolistic competition setting and showed that bilateralism can undermine political support for multilateral trade liberalization. In a similar spirit, Krishna (1998) argued that, politically government may not support further movement towards multilateral trade liberalization upon the formation of a free trade agreement. Other economists, however, stress the beneficial effects of bilateralism. Such point was advanced earlier by Summers (1991). He argued that a smaller number of trading blocs are more likely to be able to reach liberalization than a large number of countries. Later, with the help of simulations, Riezman (2000) also identified cases in which bilateral trade agreements bring globalization. In this view, economic integration between a subset of countries may raise the incentives of outsiders to join the free trade area (See Baldwin (1996) and Ethier (1998)). Most importantly, Panagariya & Krishna (2002) demonstrated a FTA version of Ohyama-Kemp-Wan theorem[4]. This is an "existence theorem" in the context of FTAs implying the possibility of realization of global free trade through FTA expansion. This paper is in a similar vein with the latter view, empasizing the positive role of bilateralism in facilitating global free trade. The novel feature of this paper lies in its application of network approach on the study of trading regime. Some recent papers, Belleflamme and Bloch (2004), Goyal and Joshi (2006), Furusawa and Konishi (2005, 2007), also use network formation games to examine closely related questions. Their analysis is based on the assumption that countries behave myopically. With the new solution concept, *pairwise farsighted stable set,* proposed by Herings, Mauleon and Vannetelbosch (2009), we examine the formation of free trade as a network formation game with forward-looking countries. Our purpose is to see whether the expansion of bilateral links is likely to continue when all countries are farsighted. In this dynamic formation process, each acting countries' decisions are guided by a welfare comparison between current networks and the end network. This dynamic process ends at some networks in a pairwise farsightedly stable set.

This paper take the bilateral free trade agreements as the foundation for global free trade.

[4]The early prominent result in "dynamic" process of free trade is called Ohyama-Kemp-Wan theorem established by Ohyama (1972) and Kemp & Wan (1976). This theorem indicated that a Pareto-improving customs union expansion is possible to reach global free trade by adjusting external tariffs and internal transfers appropriately. This has been treated as a "possibility theorem." Please refer to the survey of Bhagwati and Panagariya (1996).

Some other economists, like Saggi and Yildiz (2007), Aghion et.al (2007), provide a model in which both bilateral and multilateral negotiations are endogenous choices. In particular, Saggi and Yildiz (2007) term these two bargaining protocols as FTA game and non-FTA game. With a symmetric three-country case, they show FTA game admits both complete network and patial connected network as stable equilibria. This result is consistent with our finding that complete network is not the unique stable outcome in an n-country symmetric case. Moreover, they consider an asymmetric three-country case and conclude the FTA game is necessary to achieve global free trade. By contrast, this paper does not take asymmetry into account. Definitely, extending symmetry to asymmetry to see the role of FTA game in facilitating world trading system is our future work.

This paper is structured as follows. In section 2, the basic model is introduced and the notion of stability and efficiency are defined. Section 3 examines which free trade network is socially efficient. Section 4 shows that the complete global free trade is pairwise farsightedly stable, while section 5 concludes the discussion.

2 The basic model

Following Goyal & Joshi (2006), we consider a setting with n countries. Each country has a single firm. All firms produce a homogenous good and compete in each country's market as Cournot oligopolists. All countries are able to negotiate bilateral free trade agreements. When a bilateral link is established between two countries, these two countries have a tariff-free access to each other's market. Otherwise they impose a non-zero tariff rate on imports to protect domestic firm. Then given these optimal tariffs set by each country, each firm chooses optimal quantities to sell in domestic market and foreign markets. The equilibrium tariff rates and quantities are unique for a given network structure.

The collection of bilateral free trade agreements consists of a free trade network. Precisely, denote $I = \{1, 2, ...n\}$ the set of indexed symmetric countries, let a binary variable $g_{ij} \in \{0, 1\}$ define the relation between country i and j. $g_{ij} = 1$ implies a bilateral FTA between country i and j, and $g_{ij} = 0$ indicates no bilateral FTA between them. Therefore, a network $g = \left\{(g_{ij})_{i,j \in I}\right\}$ is simply a collection of pairwise links. We denote $g+ij$ the network structure obtained by adding link ij to the existing network g. Similarly, let $g-ij$ denote the network obtained by deleting link ij from g. If $g' = g + ij$ or $g' = g - ij$, then g and g' are said to be adjacent. We adopt the convention that $g_{ii} = 1 \ \forall i$. Denote $I_i(g) = \{j \in g : g_{ij} = 1\}$ the set of players with whom i has a bilateral link in network g, including country i, then $\eta_i(g)$ is the cardinality of $I_i(g)$. A *component* of a network g is a subset c of g such that no $i \in c$ is linked outside c and all countries in c are directly or indirectly linked. Given a graph, a component is a regional trade agreement (RTA). A RTA can take a form of combination of bilateral FTAs, in which member countries are directly or indirectly linked, or a form of custom unions (CU), in which member countries are completely connected. Let $C(g)$ denote all the components of g. For each $c \in C(g)$, denote $|c|$ the size of c, representing the number of countries in component c. Let G be the set of all possible network structures over I. There are two special elements in G, the empty network g^0 with $g_{ij} = 0 \ \forall i, j \in I$, and the complete network $\tilde{g}$ with $g_{ij} = 1 \ \forall i, j \in I$.

Once the network is established, countries make decisions on tariff rates. Let $T^i_j(g)$ be a tariff rate faced by firm i in country j in network g, $T_j^i(g) = T_i^j(g) = 0$ if $g_{ij} = 1$, and $T_j^i(g), T_i^j(g) > 0$ if $g_{ij} = 0$. Each country has one firm producing a homogenous tradeable good. Given the existing network g and the tariff rates, all firms compete in a Cournot manner in each country's independent market. Assume all countries are of the same size. In the market of country $i \in I$, all firms face an identical inverse linear demand, $P_i = \alpha - Q_i$. Here $\alpha > 0$, $Q_i = \sum_{j \in I} Q_i^j$, representing aggregate output in country i, and Q_i^j is the output level of firm j in country i. Moreover, assume all firms have a constant and identical marginal cost $\gamma > 0$, where $\alpha > \gamma$. Therefore, firms in each country choose how much to produce for domestic market and how much to export to foreign countries.

When a firm decides its export level to a foreign country, the existing network structure g and tariff rate have a bearing on its decision. With respect to the trade relation between two countries, there are two kinds of maximization problems.

If $g_{ij} = 1$, the maximization problem of firm j in country i is given by

$$\max_{Q_i^j(g)} \left(p - (\eta_i(g) - 1) Q_i^{j'}(g) - (n - \eta_i(g)) Q_i^k(g) - Q_i^j(g) \right) Q_i^j(g) - \gamma Q_i^j(g)$$

If $g_{ik} = 0$, firm k has to pay tariff imposed by country i. Since all countries are ex-ante symmetric, $Q_i^k(g) = Q_i^l(g)$ for all $k, l \in I \backslash I_i(g)$, therefore, $T_i^k = T_i$ for all $k \in I \backslash I_i(g)$. The maximization problem of firm k is

$$\max_{Q_i^k(g)} \left(p - \eta_i(g) Q_i^j(g) - (n - \eta_i(g) - 1) Q_i^{k'}(g) - Q_i^k(g) \right) Q_i^k(g) - \gamma Q_i^k(g) - T_i Q_i^k(g)$$

After standard derivation, the Cournot equilibrium outputs of a FTA country and a non-FTA country of country i are given respectively:

$$Q_i^j(g) = \frac{(\alpha - \gamma) + (n - \eta_i(g)) T_i(g)}{n+1}, \quad j \in I_i(g) \tag{1}$$

$$Q_i^k(g) = \frac{(\alpha - \gamma) - (\eta_i(g) + 1) T_i(g)}{n+1}, \quad k \in I \backslash I_i(g) \tag{2}$$

The welfare of country i is defined as the sum of consumer surplus, firm's profits and tariff revenues.

$$\begin{aligned} S_i(g) &= \frac{1}{2} Q_i^2(g) + \left[(P_i(g) - \gamma) Q_i^i + \sum_{j \neq i} \left(P_j(g) - \gamma - T_j^i(g) \right) Q_j^i(g) \right] \\ &\quad + \sum_{j \neq i} T_i^j(g) Q_i^j(g) \end{aligned} \tag{3}$$

Substituting (1) and (2) into (3) yields social welfare function of country i, which is a function of tariff rate $T_i(g)$:

$$
\begin{aligned}
(4) \qquad S_i(g) &= \frac{1}{2}\left[\frac{n(\alpha-\gamma)-(n-\eta_i(g))\,T_i(g)}{n+1}\right]^2 \\
&+ \sum_{j:g_{ij}=1}\left[\frac{(\alpha-\gamma)+\left(n-\eta_j(g)\right)T_j(g)}{n+1}\right]^2 \\
&+ \sum_{k:g_{ik}=0}\left[\frac{(\alpha-\gamma)-(\eta_k(g)+1)\,T_k(g)}{n+1}\right]^2 \\
&+ (n-\eta_i(g))\,T_i(g)\left[\frac{(\alpha-\gamma)-(\eta_i(g)+1)\,T_i(g)}{n+1}\right]
\end{aligned}
$$

To maximize (4), country i chooses the optimal tariff rate, this yields

$$
(5) \qquad T_i^*(g) = \frac{3(\alpha-\gamma)}{\eta_i(g)(2n+5)-(n-2)}
$$

Then substituting (5) into (4), each country's social welfare is determined by the network g

$$
\begin{aligned}
(6) \qquad S_i(g) &= \frac{[\eta_i(g)(2n+1)-(n-4)]}{2[\eta_i(g)(2n+5)-(n-2)]}(\alpha-\gamma)^2 \\
&+ \sum_{j\neq i:g_{ij}=1}\left[\frac{2\left(\eta_j(g)+1\right)}{\eta_j(g)(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \\
&+ \sum_{k:g_{ik}=0}\left[\frac{(2\eta_k(g)-1)}{\eta_k(g)(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2
\end{aligned}
$$

Welfare function (6) is a mapping from the set of network structures G into the vector of individual payoffs. In particular, given the network g, country i's social welfare is a function of three arguments: the number of links of country i's own, $\eta_i(g)$; the number of links of its connected countries, $\eta_j(g)$; and the number of links of its unconnected countries, $\eta_k(g)$.

3 Efficient and Stable networks

The paper considers two related issues. The first issue is to determine the efficient network structures, while the second one is concerned with whether such efficient network structures will be formed when all forward looking countries establish their bilateral free trade agreements voluntarily. The notions of efficiency and stability are provided in the following.

3.1 Strongly efficient networks

Given different valuation structures, one might consider different notions of efficiency for networks. For instance, efficiency could correspond to maximizing aggregate payoff, or it could describe Pareto optimality under a particular allocation rule. In our setting, we concentrate on the notion which maximizes global welfare. The welfare of country i is defined as $S_i(g)$, consequently, for any network g, global welfare is denoted by the sum of n countries' individual social welfare: $v(g) = \sum_{i \in I} S_i(g)$. A free trade network g is *strongly efficient* if $v(g) \geq v(g')$ for all $g' \in G$.

3.2 Pairwise farsightedly stable networks

To predict which networks may be formed among farsighted countries, we adopt a new solution concept, *the pairwise farsightedly stable set*, introduced by Herings, Mauleon and Vannetelbosch (2009). The notion of *pairwise stability* introduced by Jackson & Wolinsky (1996) is widely employed in the existing literature. The basic idea behind this notion is that countries do not benefit from altering current network structures either unilaterally or bilaterally. However, pairwise stability is a weak definition in the sense that it assumes countries add or delete links myopically. Under this myopic behavior, it is possible that a country suffers from deleting a link, but this deletion leads another country to add another link which in turn leaves the first country better off relative to the starting position. If a forward looking country foresaw this, it might choose to remove the link to initiate the process. Unfortunately, this consideration is not taken into account in the notion of pairwise stability. To fix this problem, Herings, Mauleon and Vannetelbosch (2009) proposed a *farsightedly improving path. A farsightedly improving path* is a sequence of networks that can emerge when countries form or sever links based on the improvement the end network offers relative to current network.[5] The formal definition of a farsightedly improving path is:

Definition 1 *A farsightedly improving path from a free trade network g to another free trade network g' is a finite sequence of graphs $g_1, g_2, ..., g_K$ with $g_1 = g$ and $g_K = g'$ such that for any $k \in \{1, ...K-1\}$ either:*

- $g_{k+1} = g_k - ij$ *for some* $i,\ j$ *such that* $S_i(g_K) > S_i(g_k)$, *or* $S_j(g_K) > S_j(g_k)$
- $g_{k+1} = g_k + ij$ *for some* $i,\ j$ *such that* $S_i(g_K) > S_i(g_k)$ *and* $S_j(g_K) \geq S_j(g_k)$.

Along a farsightedly improving path, if an existing FTA link is deleted, at least one of the countries involved strictly prefers the end network. If a bilateral FTA is established, then these two countries involved must both prefer the end network, and at least one strictly prefers the end network. In the language of Chwe (1994), a farsightedly improving path implies that g' indirectly dominates g. The concept of indirect domination refers to payoff comparison between each intermediate state in the farsightedly improving path and the final outcome g'. It is in this sense that this new notion of network stability incorporates farsightedness.

If g can get to g' through a farsightedly improving path, we write $g \to g'$. $F(g) = \{g' \in G | g \to g'\}$ is the set of networks that can be achieved through a farsightedly improving path from g. After defining a farsightedly improving path, the notion of pairwise farsightedly stable sets follows,

[5] Please refer to Herings, Mauleon and Vannetelbosch (2009) for more information.

Definition 2 *A set of free trade networks $G^* \subseteq G$ is pairwise farsightedly stable if*

- *1)* $\forall g \in G^*$,
  - *1a)* $\forall ij \notin g$ *such that* $g+ij \notin G^*$, $\exists g' \in G^* \cap F(g+ij)$ *such that* $(S_i(g'), S_j(g')) = (S_i(g), S_j(g))$ *or* $S_i(g') < S_i(g)$ *or* $S_j(g') < S_j(g)$,
  - *1b)* $\forall ij \in g$ *such that* $g-ij \notin G^*$, $\exists g', g'' \in G^* \cap F(g-ij)$ *such that* $S_i(g') \leq S_i(g)$ *and* $S_j(g'') \leq S_j(g)$,
- *2)* $\forall g' \notin G^*$ *we have* $g \in F(g')$ *for some* $g \in G^*$,
- *3)* $\nexists G' \subsetneq G^*$ *such that* G' *satisfies* (ia),(ib) *and* (ii)

Part (1) guarantees that all free trade networks in the stable set are immune to credible deviation. Condition ($1a$) describes the case that adding a link to g makes both countries end up being worse off or equally well off. Condition ($1b$) describes the case that cutting a link makes at least one country worse off or equally well off. Moreover, $g' \in G^*$, which warrants g' to be a credible threat. Part (2) requires any network outside the stable set can get to some network in the stable set through a farsightedly improving path. This condition implies non-emptiness of the stable set. Since the set G satisfies ($1a$), ($1b$) and (2), the minimality condition is required, which is Part (3).

4 Strongly efficient free trade networks

The following proposition shows the nature of efficient networks.

Proposition 3 *The complete global free trade network $\tilde{g}$ is the unique strongly efficient network.*

Proof. The global welfare is the sum of each country's welfare, i.e., $v(g) = \sum_{i \in I} S_i(g)$. After standard derivation, the global welfare can be rewritten as

$$
\begin{aligned}
(7) \qquad & v(g) \\
= & \sum_{i \in I} \frac{\left[\left(4n^2+12n-3\right)\eta_i(g)^2 - \left(4n^2-6n-6\right)\eta_i(g) + n^2 - 6n\right]}{2\left[\eta_i(g)(2n+5) - (n-2)\right]^2}(\alpha-\gamma)^2 \\
& + \sum_{i \in I} \sum_{j: g_{ij}=1} \left[\frac{2\left(\eta_j(g)+1\right)}{\eta_j(g)(2n+5)-(n-2)}\right]^2 (\alpha-\gamma)^2 \\
& + \sum_{i \in I} \sum_{k: g_{ik}=0} \left[\frac{\left(2\eta_k(g)-1\right)}{\eta_k(g)(2n+5)-(n-2)}\right]^2 (\alpha-\gamma)^2
\end{aligned}
$$

The expression above is decomposed of three parts: consumers surplus, producer surplus of each firm and tariff revenues. It is convenient to express the last two terms in a different way, the second term can be shown as the sum of producer surplus generated in each of the markets, while the third term can be expressed as the sum of tariff revenues generated in each country. Thus we can rewrite the global welfare as:

$$
\begin{aligned}
(8) \qquad & v(g) \\
= & \sum_{i\in I} \frac{\left[(4n^2+12n-3)\,\eta_i(g)^2 - (4n^2-6n-6)\,\eta_i(g) + n^2 - 6n \right]}{2\left[\eta_i(g)(2n+5)-(n-2)\right]^2} (\alpha-\gamma)^2 \\
& + \sum_{i\in I} \eta_i(g) \left[\frac{2(\eta_i(g)+1)}{\eta_i(g)(2n+5)-(n-2)} \right]^2 (\alpha-\gamma)^2 \\
& + \sum_{i\in I} (n-\eta_i(g)) \left[\frac{(2\eta_i(g)-1)}{\eta_i(g)(2n+5)-(n-2)} \right]^2 (\alpha-\gamma)^2
\end{aligned}
$$

in the complete network, the welfare generated in every country is the same, it is given by $\hat{S}(\tilde{g})$:

$$\hat{S}(\tilde{g}) = \frac{3n^2}{2(n+1)^2} (\alpha-\gamma)^2$$

in an arbitrary network g, the welfare generated by country i is denoted by $\hat{S}_i(g)$, which is:

$$\hat{S}_i(g) = \frac{\left[(2n+3)\eta_i(g) - n\right]\left[(2n+7)\,\eta_i(g) - (n-4)\right]}{2\left[\eta_i(g)(2n+5)-(n-2)\right]^2} (\alpha-\gamma)^2$$

We can show that $\hat{S}_i(g)$ is an increasing function of η_i by taking the first derivative with respect to η_i. $\frac{\partial \hat{S}_i}{\partial \eta_i} = \frac{12(\eta_i+1)(n+1)}{(\eta_i(2n+5)-(n-2))^3} (\alpha-\gamma)^2 > 0$. So when $\eta_i = n$, i.e., in the complete network, $\hat{S}_i$ achieves the maximal value, therefore, the complete network is uniquely efficient. ■

Corollary 4 *Given an arbitrary network, total value of the network is increasing with the number of free trade links.*

Proof. Given an arbitrary network g, suppose $ij \notin g$, when country i and j link together, the change on total welfare is

$$v(g+ij) - v(g) = \hat{S}_i(g+ij) - \hat{S}_i(g) + \hat{S}_j(g+ij) - \hat{S}_j(g).$$

We know $\hat{S}_i(g)$ is an increasing function of η_i, so the difference must have a positive sign. It means that given the network, the total welfare is monotonically increasing with the link addition. ■

5 Farsightedly stable free trade networks

In this section we consider strategically stable networks with forward-looking countries. The solution concept of *pairwise farsighted stability* is adopted to predict the outcome of a free trade network formation game. We have two main findings: First, the complete global free trade network is pairwise farsightedly stable. Second, the complete network is not a unique stable network.

The following results due to Herings, Mauleon and Vannetelbosch (2009) will be useful in characterizing the pairwise farsightedly stable free trade networks.

Theorem 5 (Herings et al.) *A pairwise farsightedly stable set of networks exists.*

Theorem 6 (Herings et al.) *The set $\{g\}$ is a pairwise farsightedly stable set if and only if for every $g' \in G\backslash\{g\}$, we have $g \in F(g')$.*

Corollary 7 (Herings et al.) *The set $\{g\}$ is the unique pairwise farsightedly stable set if and only if for every $g' \in G\backslash\{g\}$ we have $g \in F(g')$ and $F(g) = \emptyset$.*

Theorem 5 assures us that a farsightedly stable set of free trade networks always exists. Theorem 6 characterizes a single-valued farsightedly stable set. By applying Theorem 6, we obtain the following result concerning the complete global free trade network $\tilde{g}$,

Proposition 8 *$\{\tilde{g}\}$ is pairwise farsightedly stable.*

Proof. A formal proof is provided in Appendix due to the space constraint. The main idea is sketched in the following: According to Theorem 6, for $\tilde{g}$ to be pairwise farsightedly stable, it is needed to prove $\tilde{g} \in F(g), \forall g \in G\backslash\{\tilde{g}\}$. The proof is fulfilled in the following three steps:

- Step 1: Construct a sequence $P = g_1, g_2, ..., g_k, ..., g_K$ such that $g_1 = \mathring{g}$ and $g_K = \tilde{g}$. Each network on such a sequence has one more link than the previous one.

- Step 2: Prove this sequence P is a farsightedly improving path. This proof is given in Lemma 10. It is worth noticing that this sequence P is more than a farsightedly improving path requires. According to the definition, link addition requires both of the involved agents prefer the end network and at least one of them strictly prefers. On the sequence P, when two acting countries make a decision to forge a link between them, both of them strictly prefer the end network.

- Step 3: With the farsightedly improving path P from the empty network $\mathring{g}$ to the complete network $\tilde{g}$, it remains to show that $g \to \tilde{g}, \forall g \in G\backslash P$. To this end, in Lemma 11, we show that for any network g not on P, there exists at least one country i in network g satisfying $2 \leq \eta_i(g) \leq n$ and $S_i(g) < S(\tilde{g})$.[6] Thus, country i, with more than 2 links, prefers the welfare in the complete network and would like to cut its link in anticipating the complete network as the end network. Keep this in mind, starting from any network g which is not on the sequence P, we can always find a farsightedly improving path $g'_1, g'_2, ..., g'_l, g_k, ..., g_K$, where $g = g'_1$ and $g_K = \tilde{g}$. In the first half of this sequence, $g'_1, g'_2, ..., g'_l$, each network has one less link than the previous one, and g'_l is the network which differs by one link from g_k. The second half of this sequence, $g_k, ..., g_K$,

[6] In complete network $\tilde{g}$, all countries are symmetric with the same link structure. Therefore, all countries have the same welfare level. We denote the welfare of a representative country by $S(\tilde{g})$.

network with 10 countries	S_1
$\tilde{g}$	$0.495868\,(\alpha-\gamma)^2$
$g_1=\tilde{g}-12$	$0.493912\,(\alpha-\gamma)^2$
$g_2=g_1-13$	$0.492001\,(\alpha-\gamma)^2$
$g_3=g_2-14$	$0.490154\,(\alpha-\gamma)^2$
$g_4=g_3-15$	$0.488407\,(\alpha-\gamma)^2$
$g_5=g_4-16$	$0.486821\,(\alpha-\gamma)^2$
$g_6=g_5-17$	$0.48553\,(\alpha-\gamma)^2$
$g_7=g_6-18$	$0.484863\,(\alpha-\gamma)^2$
$g_8=g_7-19$	$0.485934\,(\alpha-\gamma)^2$
$g'=g_8-110$	$0.496412\,(\alpha-\gamma)^2$

Table 1: A farsightedly improving path

is also the second half of the constructed path P starting from $\mathring{g}$ to $\tilde{g}$. Along such a farsightedly improving path, for any network on the the first half of the sequence, the acting player is a country who prefers the complete network and would like to cut its link until network evolves to some point on the sequence P, then the path comes to the second part, the sequence converges to P until the end network $\tilde{g}$.

■

Proposition 8 shows that the complete global free trade network is pairwise farsightedly stable. This result implies that, given any trading regime different from global free trade, there always exists a farsightedly improving path leading it to the complete trading network. This nice result is due to the assumption of farsighted behavior of all countries. When a country forms or deletes a trade agreement, it makes decision based on the improvement of the complete network offers relative to the current network rather than the adjacent network offers. With farsighted behavior, the complete global free trade is sustainable.

Proposition 9 demonstrates the non-uniqueness of network $\tilde{g}$.

Proposition 9 $\{\tilde{g}\}$ *is not the unique pairwise farsightedly stable set.*

Proof. According to Corollary 7, if for every $g \in G\backslash\{\tilde{g}\}$, we have $\tilde{g} \in F(g)$ and $F(\tilde{g}) = \emptyset$, then $\{\tilde{g}\}$ is the unique pairwise farsightedly stable set. Proposition 8 has shown that for every $g \in G\backslash\{\tilde{g}\}$, $\tilde{g} \in F(g)$. To prove $\{\tilde{g}\}$ is not the unique pairwise farsightedly stable set, it suffices to show that $F(\tilde{g}) \neq \emptyset$. We demonstrate it by providing a counter example. Consider the complete network $\tilde{g}$ with $n = 10$, we claim there exists a farsighted improving path from $\tilde{g}$ to g', where g' consists of one singleton and a fully connected component with the remaining 9 countries. The path and the welfare of the singleton country 1 corresponding to each network are listed in Table 1.

It is easy to check along such a sequence, country 1's welfare is less than the one in g', so $\tilde{g} \to g'$. Hence $F(\tilde{g}) \neq \emptyset$, $\{\tilde{g}\}$ is not the unique pairwise farsightedly stable set. ■

Proposition 9 claims that the complete global free trade is not the unique stable network, therefore, there exists other stable trading networks different from the complete network. However, it is hard to find those stable networks because the process of locating them is complex. One thing can be sure is that all remaining stable networks are not strongly efficient. This is because any other stable networks has less number of free trade links than the complete one. Then, according to Corollary 4, these stable networks have less total welfare than the complete network, the unique strongly efficient trading regime.

6 Conclusion

The main contribution of this paper is the discussion of "dynamic" process of FTAs network formation. Taking the perspective of FTAs as a cornerstone of free trade, this paper brings into focus on the incentives for countries to form bilateral free trade agreements, especially concerns the strategic stability of network structure in a forward-looking sense.

The present paper has investigated whether bilateral free trade agreements are compatible with global free trade when all countries are forward-looking. Building on Goyal and Joshi (2006)'s model of free trade networks, we engage the notion of pairwise farsightedly stability and attempt to predict which network may be formed in this free trade network formation game. It has been found that when all countries are symmetric and forward-looking, the complete global free trade network is pairwise farsightedly stable. As a result, bilateral free trade agreements are "building blocs" towards the world trading system. This finding is consistent with earlier studies conducted by Goyal and Joshi (2006) and Furusawa and Konishi (2007). In addition, it has been shown that the complete network is uniquely strongly efficient. Therefore, the individual incentives and social incentive generally coincide. Unfortunately, the complete network is not a unique stable network, some other farsightedly pairwise stable networks might emerge. The policy implication is that free trade network formation process is "path dependent", member countries of the WTO need to put more focus on the policy which encourages all countries to follow the "right path" leading to the global free trade.

It should be noted that even some degree of farsightedness is accommodated in our network formation model, the sort of "introspective farsightedness" considered here is different from a description of actual play along an explicitly dynamic path (such a dynamic path is considered in Dutta et al. 2005). Therefore, an exploration of explicit dynamic network formation process is left for future work. Also, we have assumed symmetry in this n-country model. Many aspects, for example, like asymmetric countries and weights on different components of welfare, can be incorporated to improve the model. It would be interesting to test the robustness of our results in such richer environments.

7 Appendix

Proof of Proposition 8

Step1: Construction. We construct a sequence P from $\mathring{g}$ to $\tilde{g}$ through an iterative procedure. As assumed, there are n indexed countries. In the first round, starting from country 1, each country set up a link to the country next to it according to the index. For example, country 1 links to country 2, country 2 links to country 3,..., country i links to country $i+1$, etc. Finally, country n links to country 1 such that a circle is achieved. In the second round, according to the index, each country set up a link to the country two steps away from it, i.e, country 1 links to country 3, country 2 links to country 4,.... In particular, country $n-1$ links to country 1 and country n links to country 2. Follow the same fashion, in the qth round, country i links to country $i+q$, which is q steps away from country i. At the end of qth round, a symmetric network is arrived, where each country has $2q+1$ links.

If n is odd, the last round is $\frac{n-1}{2}th$ round. Country i links to country $i+\frac{n-1}{2}$. At the end of the last round, the complete network $\tilde{g}$ is achieved.

If n is even, the second to last round is $\left(\frac{n}{2}-1\right)th$ round. The network at the end of this round is a symmetric network with each country having $n-1$ links. The last round is $\frac{n}{2}th$ round. In this round, countries link diagonally. In the end, the complete network is achieved.

Step 2: **Proof of a farsightedly improving path**. After constructing the sequence P, we proceed to prove P is a farsightedly improving path in the following Lemma.

Lemma 10 *The sequence P from $\mathring{g}$ to $\tilde{g}$ is a farsightedly improving path.*

Proof. n could be odd or even, then we discuss these two cases respectively:

Case 1: when n is odd. Given an arbitrary round q $(1 \leq q \leq \frac{n-1}{2})$, country 1 will link with country $q+1$ who is q units away from it. Country 2 sets up a link with country $q+2$,..., and so on. It is needed to show that at qth round, two acting countries who are about to link have less welfare than the one in complete network $\tilde{g}$. The proof takes the following two steps: In the first step, general welfare functions for any two acting countries who are about to link are determined; In the second step, we locate the country with the largest welfare level in all rounds and show that it still has less welfare than $S(\tilde{g})$.

In the first step, we characterize the welfare function for all acting countries. From (6), we know welfare is determined by the current graph, i.e., the link structure. So we need to characterize the link structure first. To simplify analysis, we divide countries into two groups. Group 1 consists of the proposing countries, and group 2 consists of the responding countries. For example, at qth round, country 1 links to country $q+1$. We categorize country 1 into group 1 as proposing country, and country $q+1$ into group 2 as responding country.

For an arbitrary round q, Table 2 to Table 5 list each country's link structure. In these tables, the first column is country's index. The second column indicates the country's own number of links, η_i. The third column lists the number of linked countries (the first entry) and the corresponding links for each linked country (the second entry). The fourth column shows the number of non-linked countries (the first entry) and their own number of links (the second entry). Take country 1 as an example. At qth round, she has $2q-1$ links. Meanwhile, her connected countries are of two kinds: one kind has $2q-1$ links and there are $2(q-1)$ of them, and the other kind has $2q$ links, but there are 0 of them. Her unconnected countries has $2q-1$ links and there are $n-2q+1$ of them.

After a series of comparisons, we find when $q=\frac{n-1}{2}$, country $\frac{n+1}{2}$ achieves the maximal

index of i	η_i	η_j	η_k
$i \in [1, q]$	$2q-1$	$2(q-i); 2q-1$ $2(i-1); 2q$	$n-2q+1; 2q-1$
$i \in [q+1, 2q]$	$2q$	$3q-i; 2q$ $i-q-1; 2q+1$	$n-q-i+1; 2q-1$ $i-q-1; 2q$
$i \in [2q+1, n-q]$	$2q$	$q-1; 2q$ $q;2q+1$	$n-q-i+1; 2q-1$ $q;2q$ $i-2q-1; 2q+1$
$i \in [n-q+1, n]$	$2q$	$n-i; 2q$ $2q+i-n-1; 2q+1$	$n-i+1; 2q$ $i-2q-1; 2q+1$

Table 2: when q is between 1 and n/3, each country's structure of links in group 1

index of i	η_i	η_j	η_k
$i \in [q+1, 2q]$	$2q-1$	$3q-i-1; 2q-1$ $i-q-1; 2q$	$n-q-i+2; 2q-1$ $i-q-1; 2q$
$i \in [2q+1, n-q]$	$2q-1$	$q-1; 2q-1$ $q-1; 2q$	$n-q-i+1; 2q-1$ $q+1; 2q$ $i-2q-1; 2q+1$
$i \in [n-q+1, n]$	$2q-1$	$n-i; 2q-1$ $2q+i-n-2; 2q$	$n-i+2; 2q$ $i-2q-1; 2q+1$
$i \in [1, q]$	$2q$	$2(q-i); 2q$ $2i-1; 2q+1$	$1; 2q$ $n-2q-1; 2q+1$

Table 3: when q is between 1 and n/3, country's structure of links in group 2

index of i	η_i	η_j	η_k
$i \in [1, q]$	$2q-1$	$2(q-i); 2q-1$ $2(i-1); 2q$	$n-2q+1; 2q-1$
$i \in [q+1, n-q]$	$2q$	$3q-i; 2q$ $i-q-1; 2q+1$	$n-q-i+1; 2q-1$ $i-q-1; 2q$
$i \in [n-q+1, 2q]$	$2q$	$n+2q-2i+1; 2q$ $2i-n-2; 2q+1$	$n-2q; 2q$
$i \in [1, q]$	$2q$	$2(q-i);2q$ $2i-1; 2q+1$	$2q$ $n-2q-1; 2q+1$

Table 4: when q is between n/3 and n/2, country's structure of links in group 1

index of i	η_i	η_j	η_k
$i\in[q+1,n-q]$	$2q-1$	$3q-i-1;2q-1$ $i-q-1;2q$	$n-q-i+2;2q-1$ $i-q-1;2q$
$i\in[n-q+1,2q]$	$2q-1$	$n-2i+2q;2q-1$ $2i-n-2;2q$	$1;2q-1$ $n-2q;2q$
$i\in[2q+1,n]$	$2q-1$	$n-i;2q-1$ $2q+i-n-2;2q$	$n-i+2;2q$ $i-2q-1;2q+1$
$i\in[1,q]$	$2q$	$2(q-i);2q$ $2i-1;2q+1$	$2q$ $n-2q-1;2q+1$

Table 5: when q is between n/3 and n/2, country's structure of links in group 2

value.[7] We denote this maximal value by $S_{\max}$ and compare it to $S(\tilde{g})$

$$\begin{aligned}&S(\tilde{g})-S_{\max}\\=\;&\frac{3\Theta(n)}{4(2n^5+4n^4-9n^3-19n^2+4n+12)^2}(\alpha-\gamma)^2\end{aligned}$$

where $\Theta(n)=8n^6+32n^5-58n^4-208n^3+65n^2+266n+21$. To prove $S(\tilde{g})-S_{\max}>0$. It's sufficient to show that $\Theta(n)>0$. By taking derivative with respect to n, we have

$$\begin{aligned}\Theta'(n)&=48n^5+160n^4-232n^3-624n^2+130n+266\\\Theta''(n)&=240n^4+640n^3-696n^2-1248n+130\\\Theta'''(n)&=960n^3+1920n^2-1392n-1248\\\Theta^4(n)&=2880n^2+3840n-1392\end{aligned}$$

So it is easy to get $\Theta^4(n)>0$ when $n\geq 3$, therefore $\Theta'''(n)$ is an increasing function. Keep checking $\Theta'''(n)|_{n=3}=37\,776>0$, so $\Theta''(n)$ is an increasing function. Again $\Theta''(n)|_{n=3}=26\,842>0$, so $\Theta'(n)$ is also an increasing function. Then, $\Theta'(n)|_{n=3}=13\,400>0$ and $\Theta(n)$ is an increasing function, now $\Theta(n)|_{n=3}=4698>0$. So the largest value is still less than the complete one. Therefore we conclude that in case 1 with n being odd number, all acting countries have less welfare than the one in complete network.

Case 2: If n is even, then the sequence before the last round is the same as the one with $n-1$ countries. Until the second to last round, the maximal value is $S_{\max}$. Now look for the maximal value in $\frac{n}{2}th$ round and compare it to $S_{\max}$.When $q=\frac{n}{2}$, countries link diagonally. All countries' own number of links are from $n-1$ to n, thus any two acting countries are symmetric. They have the same welfare level. In the last round $q=\frac{n}{2}$, when country 1 is about to link with country $\frac{n}{2}+1$, it has the largest welfare, denoted by S_0. Take the difference between S_1 and $S_{\max}$,

$$\begin{aligned}&S_1-S_{\max}\\=\;&\frac{3\Lambda(n)}{2(n-2)^2(n+2)^2(-2n+2n^2-13)^2(2n+2n^2-3)^2}\end{aligned}$$

[7] Due to the space constraint, we omit the comparisons, the detailed calculations can be provided upon request.

where $\Lambda(n) = -6754n - 5369n^2 + 5352n^3 + 2668n^4 - 1168n^5 - 420n^6 + 80n^7 + 16n^8 + 3774$. To show $\Lambda(n) > 0$, we need to check whether $\Lambda(n)$ is monotonically increasing,

$$\begin{aligned}
\Lambda'(n) &= 128n^7 + 560n^6 - 2520n^5 - 5840n^4 + 10\,672n^3 + 16\,056n^2 - 10\,738n - 6754 \\
\Lambda''(n) &= 896n^6 + 3360n^5 - 12\,600n^4 - 23\,360n^3 + 32\,016n^2 + 32\,112n - 10\,738 \\
\Lambda'''(n) &= 5376n^5 + 16\,800n^4 - 50\,400n^3 - 70\,080n^2 + 64\,032n + 32\,112 \\
\Lambda^4(n) &= 26\,880n^4 + 67\,200n^3 - 151\,200n^2 - 140\,160n + 64\,032 \\
\Lambda^5(n) &= 107\,520n^3 + 201\,600n^2 - 302\,400n - 140\,160 \\
\Lambda^6(n) &= 322\,560n^2 + 403\,200n - 302\,400
\end{aligned}$$

It is easy to observe that $\Lambda^6(n) > 0$ when $n \geq 3$, then $\Lambda^5(n)$ is increasing; The smallest even number n can take is 4, so we check $\Lambda^5(n)|_{n=4} = 8757\,120 > 0$, then $\Lambda^4(n)$ is increasing; Again $\Lambda^4(n)|_{n=4} = 8266\,272 > 0$, so $\Lambda'''(n)$ is increasing; $\Lambda'''(n)|_{n=4} = 5747\,184 > 0$, then $\Lambda''(n)$ is increasing; $\Lambda'(n)|_{n=4} = 3019\,982 > 0$, then $\Lambda'(n)$ is increasing; $\Lambda'(n)|_{n=4} = 1205\,590 > 0$, then $\Lambda(n)$ is increasing; with $\Lambda(n)|_{n=4} = 359\,334 > 0$, we conclude so $S_1 - S_{\max} > 0$ and S_1 has the largest value. It remains to show $S_1 < S(\tilde{g})$, then

$$S(\tilde{g}) - S_1 = \frac{3\left(4n^2 + 4n - 3\right)}{2\left(-2n^3 - 4n^2 + n + 3\right)^2} > 0$$

Then in case 2 with n being even number, all acting countries has less welfare than $S(\tilde{g})$.

To summarize, in the sequence P we defined above, the acting countries always have less welfare than the one in the complete network. Hence they have incentive to add a link. The sequence P is a farsightedly improving path from $\mathring{g}$ to $\tilde{g}$. ■

Step 3: The third step is to examine the networks which are not on the sequence P. For these networks, we want to show they are able to achieve the complete network through a farsightedly improving path. It suffices to show for any network which is not on the sequence P, there always exists a country who prefers the welfare in the complete network and would like to cut its links. The following lemma demonstrates the existence of such a country.

Lemma 11 *Denote P the set of networks on the farsightedly improving path we constructed above, then $\forall g \in G \backslash P$, $\exists i$ s.t. $2 \leq \eta_i(g) \leq n$ and $S_i(g) < S(\tilde{g})$*

Proof. By negation, suppose $\exists g \in G \backslash P$, there doesn't exist country i such that $2 \leq \eta_i \leq n$ and $S_i(g) < S_i(\tilde{g})$. Then $\forall i$, if $2 \leq \eta_i \leq n$, we must have $S_i(g) \geq S(\tilde{g})$, while if $S_i(g) < S(\tilde{g})$, then $\eta_i = 1$. Denote the first kind of country type I and the second kind type II. Actually, type I is the member country of some component, and type II is a singleton. So network g must belong to one of the following cases:

- Case 1: g consists of only type I countries, i.e., $2 \leq \eta_i(g) \leq n$ and $S_i(g) \geq S(\tilde{g})$ $\forall i \in I$.
    - If all countries are symmetric,
        - If $\eta_i = n \ \ \forall i \in I$, then g is the complete network. So g is on the sequence P, contradicting our assumption.

    - $*$ If $2 \leq \eta_i < n \quad \forall i \in N$,we know complete network is the unique efficient one, so $S_i(g) < S(\tilde{g})$, $\forall i \in I$, contradicting $S_i(g) \geq S(\tilde{g})$, so this network doesn't exist.

  - If countries are not symmetric, there exist at least one country such that $S_i(g) > S_i(\tilde{g})$. Sum over all countries' welfare, yielding $\sum_{i \in N} S_i(g) > \sum_{i \in N} S(\tilde{g})$, contradicting that the complete network is the unique efficient network. This network doesn't exist either.

- Case 2: g consists of only type II countries, i.e., $S_i(g) < S(\tilde{g})$ and $\eta_i = 1$, $\forall\ i \in I$. Now g is the empty network. It is on the sequence P, contradicting the assumption.

- Case 3: g consists of both type I and type II countries. Now we are ready to prove non-existence of networks in case 3. If we can prove type II (a singleton country) does not have the lowest welfare, or even it has the lowest welfare, type I (a member country of some component) does not have the greater welfare than $S(\tilde{g})$, then case 3 does not exist. In the following part, what we do is to compare the welfare difference between a singleton country and a member country within some arbitrary component. To simplify the analysis, we consider the network g' with one singleton and one component c_{n-1} with $n-1$ countries. This specification does not affect our result since in an arbitrary network, all other components (except for the singleton and the underlying component) as non-linked parts can be cancelled out when taking the difference.
  Consider network g', The average value of component c_{n-1} can be represented as follows

$$\begin{aligned} & \frac{w(c_{n-1}, g')}{n-1} \\ = & \frac{(\alpha-\gamma)^2}{n-1} \sum_{i \in c_{n-1}} \left(\frac{(4n^2+20n+13)\eta_i^2 + (20-2n-4n^2)\eta_i + (n^2-4n-2)}{2(5\eta_i - n + 2n\eta_i + 2)^2} \right) \\ & + \left(\frac{1}{(2n+5)-(n-2)} \right)^2 (\alpha-\gamma)^2 \end{aligned}$$

  At the same time, the welfare of a singleton country can be calculated as

$$\begin{aligned} S_1(g') &= \frac{(2n+1)-(n-4)}{2[(2n+5)-(n-2)]} (\alpha-\gamma)^2 \\ &\quad + \sum_{i \in c_{n-1}} \left[\frac{(2\eta_i(g')-1)}{\eta_i(g')(2n+5)-(n-2)} \right]^2 (\alpha-\gamma)^2 \end{aligned}$$

  Take the difference between them, yielding

$$\begin{aligned} & S_1(g') - \frac{w(c_{n-1}, g')}{n-1} \\ = & \frac{(\alpha-\gamma)^2}{n-1} \sum_{i \in c_{n-1}} \frac{\Omega(\eta_i)}{(n+7)^2 (5\eta_i(g) - n + 2n\eta_i(g) + 2)^2} \end{aligned}$$

where $\Omega(\eta_i) = (6n^2 + 39n - 102)\,\eta_i^2(g) + (36 - 144n - 18n^2)\,\eta_i(g) + (12n^2 + 105n + 66)$. If $S_1(g') - \frac{w(c_{n-1},g')}{n-1} > 0$, the singleton country doesn't have the lowest welfare, case 3 does not exist. If $S_1(g') - \frac{w(c_{n-1},g')}{n-1} < 0$, singleton country may or may not be the country with the lowest welfare. In order to show the non-existence of case 3, we need to prove that a member country in the component has less welfare than complete one. Now we analyze the property of expression $\Phi(\eta_i) = \frac{\Omega(\eta_i)}{(n+7)^2(5\eta_i(g)-n+2n\eta_i(g)+2)^2}$. Take the first derivative of it with respect to η_i, yielding $\Phi'(\eta_i) = \frac{6((4n-2)\eta_i-(5n+2))}{(5\eta_i-n+2n\eta_i+2)^3} > 0$ since $\eta_i \geq 2 > \frac{(5n+2)}{(4n-2)}$. Thus $\Phi(\eta_i)$ is an increasing function of η_i.

- When $\eta_i = 5$, $\Phi(\eta_i)|_{\eta_i=5} > 0$. Since $\Phi(\eta_i)$ is an increasing function, for any $\eta_i \geq 5$, $\Phi(\eta_i) > 0$ holds always. In this case, the singleton country does not have the lowest welfare, case 3 does not exist.
- When $\eta_i = 4$, $\Phi(\eta_i)|_{\eta_i=4} > 0$ if $n \geq 5$. So when $n \geq 5, \eta_i \geq 4$ $\forall i \in c_{n-1}$,we have $\Phi(\eta_i) > 0$, and $S_1(g') - \frac{w(c_{n-1},g')}{n-1} > 0$.
- When $\eta_i = 3$, $\Phi(\eta_i)|_{\eta_i=3} = \frac{(12n^2+24n-744)}{(5n+17)^2(n+7)^2} > 0$ when $n \geq 7$. Therefore, when $n \geq 7$ and $\eta_i \geq 3$ $\forall i \in c_{n-1}$, we have $\Phi(\eta_i) > 0$. However, if $n = 4, 5, 6$ and $\eta_i \geq 3$ $\forall i \in c_{n-1}$, we need to examine them case by case.
  * If $n = 4$, and $\eta_i \geq 3$ $\forall i \in c_{n-1}$, the only case is the network with one singleton and one circle with 3 countries. It is easy to check $S_1(g') - \frac{w(c_{n-1},g')}{n-1} < 0$. Furthermore, we need to prove the welfare of the member country has less welfare than $S(\tilde{g})$. Denote the welfare of member country with 3 links by $S_{c_3}(g')$, then the difference is

$$\begin{aligned} & S(\tilde{g}) - S_{c_3}(g') \\ = & \frac{3(-368n + 1230n^2 + 288n^3 + 13n^4 - 3051)}{2(n+7)^2(5n+17)^2(n+1)^2}(\alpha-\gamma)^2 \end{aligned}$$

    when $n = 4, S(\tilde{g}) - S_{c_3}(g')|_{n=4} = \frac{110\,751}{8282\,450} > 0$. So the welfare of the member country has less welfare than $S(\tilde{g})$.
  * If $n = 5$, and $\eta_i \geq 3$ $\forall i \in c_{n-1}$, if the component c_{n-1} is symmetric, then $\Phi(\eta_i) < 0$ and $S_1(g') - \frac{w(c_{n-1},g')}{n-1} < 0$. Again we need to prove $S(\tilde{g}) - S_{c_3}(g') > 0$ when $n = 5$. Use the same logic, we can get $S(\tilde{g}) - S_{c_3}(g')|_{n=5} = \frac{9}{784} > 0$. If the component c_{n-1} is not symmetric, the only asymmetric case with $n = 5$ is shown in Figure 1. In this case, the welfare of country 2 and 5 are $0.463627(\alpha-\gamma)^2$, the welfare of country 3 and 4 are $0.487478(\alpha-\gamma)^2$, and $S(\tilde{g}) = 0.486111(\alpha-\gamma)^2$. it's easy to see country 2 and 5 prefer complete one.
  * If $n = 6$ and $\eta_i \geq 3$ $\forall i \in C_{n-1}$, similarly, if the component c_{n-1} is symmetric, then $S_1(g') - \frac{w(c_{n-1},g')}{n-1} < 0$. Again we have $S(\tilde{g}) - S_{c_3}(g')|_{n=6} > 0$. If the component c_{n-1} is not symmetric, there are some cases we need to analyze. All possible networks are illustrated in Figure 2-5 When $n = 6$,

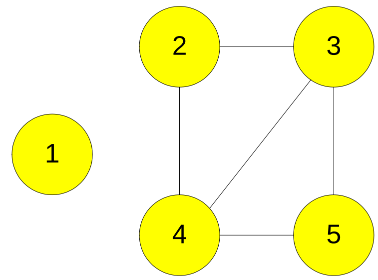

Figure 1: Asymmetric network with 5 countries

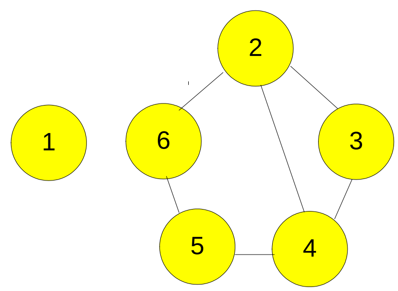

Figure 2: Asymmetric network 1 with 6 countries

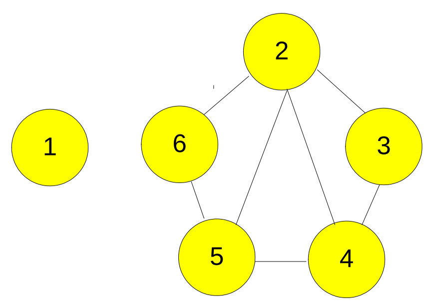

Figure 3: Asymmetric network 2 with 6 countries

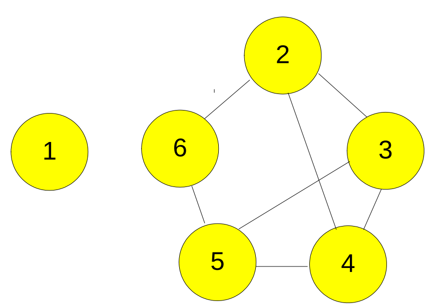

Figure 4: Asymmetric network 3 with 6 countries

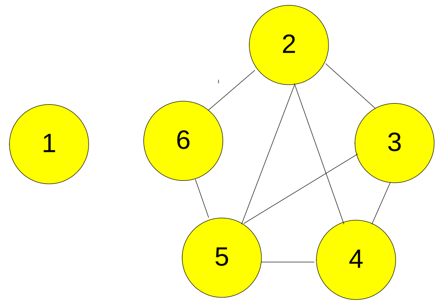

Figure 5: Asymmetric network 4 with 6 countries

there are four kinds of asymmetric networks as shown in Figure 2-5. In network 1, country 3, 5, 6 have welfare $0.476201\,(\alpha-\gamma)^2$, which is less than $S(\tilde{g}) = 0.489\,795\,91\ (\alpha-\gamma)^2$. In network 2, country 3 and 6 have $0.469176\,(\alpha-\gamma)^2$, country 4 and 5 have $0.483194\,(\alpha-\gamma)^2$, less than $S(\tilde{g}) = 0.489\,795\,91\ (\alpha-\gamma)^2$. In network 3, country 1 has $0.482246\,(\alpha-\gamma)^2$, country 2 and 5 have $0.486306\,(\alpha-\gamma)^2$, country 3 and 4 have $0.481102\,(\alpha-\gamma)^2$ and country 6 has $0.472288\,(\alpha-\gamma)^2$, it can be seen none of them has greater welfare than complete one. In the last network, country 3 and 4 have $0.476169\,(\alpha-\gamma)^2$, country 6 has $0.467356\,(\alpha-\gamma)^2$, therefore, country 3,4,6 have less welfare and prefer the complete one.

– When $\eta_i = 2$, $\Phi(\eta_i)|_{\eta_i=2} = -\frac{27n+270}{(3n+12)^2(n+7)^2} < 0$. If c_{n-1} is symmetric, then $n=3$, g' is the graph with one singleton and a linked pair. Denote the welfare of country with 2 links by $S_{c_2}(g')$, it is easy to see $S(\tilde{g}) - S_{c_2}(g') = \frac{783}{39\,200}(\alpha-\gamma)^2 > 0$. However, if $c_{n-1} \in g'$ is not symmetric, then $n > 3$. If we can show $S_{c_2}(g') < S_1(g')$, then singleton country doesn't have the lowest welfare, case 3 does not exist. Look for g' where the country with 2 links can get the maximal value. The welfare of country with 2 links is

$$\begin{aligned} S_{c_2}(g') &= \frac{2(2n+1)-(n-4)}{2[2(2n+5)-(n-2)]}(\alpha-\gamma)^2 \\ &+ \left[\frac{8}{3(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \\ &+ \left[\frac{1}{(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \\ &+ \left[\frac{(2n-5)}{(n-2)(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \\ &+ (n-4)\left[\frac{(2n-7)}{(n-3)(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \end{aligned}$$

The welfare of the singleton country is

$$\begin{aligned}
S_1(g') &= \frac{(2n+1)-(n-4)}{2\left[(2n+5)-(n-2)\right]}(\alpha-\gamma)^2 \\
&+ \left[\frac{3}{2(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \\
&+ \left[\frac{5}{3(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \\
&+ \left[\frac{(2n-5)}{(n-2)(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2 \\
&+ (n-4)\left[\frac{(2n-7)}{(n-3)(2n+5)-(n-2)}\right]^2(\alpha-\gamma)^2
\end{aligned}$$

taking difference, yielding $S_1(g') - S_{c_2}(g') = \frac{3\left(12n^4+209n^3+1173n^2+2379n+1079\right)}{(5n^3+72n^2+327n+476)^2} > 0$. Obviously, the sign of the difference is positive, so it must be the case that singleton country does not have the lowest welfare.

- To summarize the proof, we prove case 3 with both type I and type II countries does not exist, either type II country does not have the lowest welfare or when it has the lowest welfare, there exists some member country of the component with less welfare than the one in the complete network. Therefore case 3 does not exist.

- Until now, we have finished the proof of Lemma 11, for any network which is not on the constructed sequence P, there always exists a country with more than 2 links and less welfare than $S(\tilde{g})$.

■

The third step is based on Lemma 11, with this lemma, we conclude for any network, there is a farsightedly improving path leading it to the complete one.

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